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Spatial Abilities of Students and Teachers in Mathematics

1. **Dr. Noor Muhammad (Principal Author)** noormk72@gmail.com
2. **Muhammad Ghayur** muhammadghayurpet@gmail.com
3. **Dr. Muhammad Shuaib** dr.shuaib1983@gmail.com
4. **Rahmat Hussain** rahmathussain2016@gmail.com
5. **Fehmina Anjum** fanjum1989@gmail.com
6. **Muhammad Alam Khan** muhammadalam334@gmail.com

ABSTRACT

Spatial abilities are regarded as pre-requisite for solving complex mathematical problems. The present study is aimed at measuring the spatial ability among Students' and Mathematics teachers. The major objectives of the study were; (i) to investigate spatial ability of students and Mathematics teachers, (ii) to compare the spatial ability of public and private schools' students and Mathematics teachers, (iii) to compare the spatial ability of rural and urban schools' students and Mathematics teachers. On the basis of the above objectives, null hypotheses were developed and tested. A total of 400 students from sixteen schools who were studying in 9th class (Science Group) and their sixteen Mathematics teachers were taken as sample of the study. An empirical research design was used in the study. For data collection, twenty five test items were developed and observation sheets were used to measure the performance of the sample. Statistical techniques such as Mean, Standard Deviation, and t-test were applied to analyze the collected data. The study is significant for students and teachers as well as for researchers and curriculum developers. The study concluded that; urban private schools' students performed better than the other sampled students. Similarly, the Mathematics teachers performed better than the students. It was recommended that the existing curriculum should be revised to enhance the spatial ability of the students.

Keywords: Spatial Ability, Students, Mathematics teachers

INTRODUCTION

Mathematics is one of the most important subjects, which is not only necessary for academic achievement but also for everyday life. Therefore, the students must learn mathematics. Because of the importance of mathematics, how to develop students' basic mathematical skills has been a crucial issue for the educators and teachers for years. One of the desired suggestions to develop mathematical skills is to suitably emphasize and develop primary abilities such as spatial ability instead of just teaching mathematics. Spatial ability was considered to be one of the primary abilities that seem especially important in learning and doing mathematics. Spatial ability is a good predictor for mathematics performance. Further at age 13 this ability is highly associated with science subject selection at age-18. (Young, Levine, & Mix, 2018; Tosto et al., 2014). Many studies found spatial skills were positively correlated with measures of mathematics performance. Spatial abilities are claimed to be powerful tools for understanding and solving mathematics problems. It is also reported that spatial ability not only influence mathematics achievement but also it was strongly linked to achievement in science. And further, there increase in spatial ability also affects their greater achievement in science subject. The consensus among mathematics educators and researchers that visualization, or spatial ability is important because it enhances a global and intuitive view and understanding in many areas of mathematics. Research studies show that there are relationships between spatial ability and geometrical achievement. Rouadi and Husni (2014) described that to learn geometry the spatial thinking is very essential and in this regards shape recognition or visualization is the pre-requisite for this wonderful subject. Different rotations of the object should be conserved by the learner in the initial stage of the geometry.

Spatial ability is absolutely essential to living in a three-dimensional world. According to Patkin and Dayan (2012), spatial reasoning skills affect our ability to “navigate from place to place, detect an object moving towards us, estimate quantities, understand drawings and charts, and compose various items” (p. 179-180). Spatial ability is listed as one of Howard Gardner’s multiple intelligences, focusing on non-verbal cognitive abilities (Gardner, 2006). Spatial intelligence covers a wide variety of right-brain thinking and processes including visual, motor, analytic, and behavioral skills (Meadmore, Dror, & Bucks, 2009; Patkin & Dayan, 2012).

Generally, spatial ability can be divided into three categories. These categories are mental rotation (MR), spatial visualization (SV), and spatial orientation (SO).

The three categories of spatial ability play a major role in geometric thinking and understanding (Unal, Jakubowski, & Corey, 2009). Educational research regarding spatial ability dates back to the 1940s, but behavioral analysis of spatial ability has been noted earlier by research conducted in the field of psychology. Spatial reasoning skills are absolutely essential for improving mathematical comprehension and mathematical pedagogy for mathematics educators. In fact, the National Council of Teachers of Mathematics (NCTM) identified spatial ability as a central goal in high school mathematics, specifically in geometry courses (The National Council of Teachers of Mathematics, 2013). The role of spatial ability is recognized in various other subjects including chemistry, geosciences, physics, and engineering. Patkin and Dayan (2012) suggest that one’s spatial ability influences and can determine one’s level of achievement in mathematics and other related fields.

Objective of the Study

Objectives of the study were,

(i) to investigate spatial ability of students and Mathematics teachers,
(ii) to compare the spatial ability of public and private schools’ students and their Mathematics teachers,

(iii) to compare the spatial ability of rural and urban schools' students and Mathematics teachers.

Hypothesis of the Study

To achieve objective of the study, the following null hypotheses was tested;

H01: There is no significant difference between the spatial abilities of students and Mathematics teachers.

H02: There is no significant difference between the spatial abilities of rural and urban participants.

H03: There is no significant difference between the spatial abilities of public and private participants.

Significance of the Study

The study was equally significant for all secondary school students, teachers teaching Mathematics as well as researchers and curriculum developers.

Literature Review

Human thought can be divided in mathematics in two ways: they reason either verbally or reason in spatial way. In verbal reasoning human assemble the symbol in meaningful way while in spatial reasoning he uses spatial relationship for making idea between two entities. Spatial reasoning or thinking is a mental activity on the basis of which human create spatial images in his mind and through manipulation of those images he solves different theoretical and practical problems. Mostly in navigation, manipulation and designing human mind uses spatial thinking for processing information. Spatial thinking is even more important for higher mathematics and for abstract thought representation (Jones 2001, p. 55).

In simple word visualization is the process of presenting the object or event in real way. Closer examination reveals, however, that for many visualization realistic depiction is neither their function nor intention. Visualization object as any object that a student observes to assist in the learning or understanding of some topic of educational importance. A visualization object could be a picture, a schematic diagram, a computer simulation, or a video. Using visualization for object is said to be visualization. The students who use visual imagery in the absence of visualization objects is said to be introspectory. Visualization makes learning easier and concrete (Phillips, Norris, & Macnab, 2010).

Visualization can be categorized into three different categories: 1) one common usage involves the idea that visualization is something that people do-they visualize. This process is typically seen as a mental process in which certain thoughts have content that is related to- perhaps is identical to- the content of something that is seen with the eye; 2) a second sense of term visualization refers to simply to imagining that is key to the success; 3) the last is computer-generated animation are referred to as visualization (Phillips, Macnab, & Norris, 2010).

To learn geometry the spatial thinking is very essential and in this regards shape recognition or visualization is the pre-requisite for this wonderful subject. Different rotations of the object should be conserved by the learner in the initial stage of the geometry (Ruadi & Husna, 2014).

Spatial ability research has been conducted in both the areas of psychology and graphics education. In psychological research, mechanical ability and comprehension are terms found in the vocational, occupational, and interest testing areas (Aiken, 2000; Murphy & Davidshofer, 2005). In graphics education and psychology literature, various spatial ability Concepts, such as visualization and orientation, have been identified by different terms, broadly called “factors.” Carroll (1993) presented the general term “ability” as “the quality of being able to do something,” [for example physically, mentally, or spatially] that can be measured (p. 3). This measurement of ability can take the form of determining the components or factors that comprise the ability in question. In *An International Directory of Spatial Tests,* Eliot and Smith (1983) discussed spatial ability as being intricate with many unidentified associations or relationships. Carroll described numerous spatial ability factors, such as spatial relations and visualization that have been identified by researchers in the visual perception domain. Based on his review of historical research, Carroll further stated that “considerable confusion exists about the identification of factors in the domain of visual perception” due to a vague understanding of factors and variations in factor names (p. 308). From Carroll’s (1993) factor research analysis of numerous cognitive domains, the visual perception domain in particular has been cited by many graphics education researchers (Embretson, 1996; Hegarty & Waller, 2004; Miller & Bertoline, 1991). McGee (1979) stated that “historically, the identification of the Spatial [McGee’s capitalization] factor [spatial abilities as distinct from verbal abilities] has roots in the study of mechanical ability factor of visualization. aptitude” that originated in the 1920’s (p. 7). Agresti and Finlay (1997) discussed factors as variables with possibly interrelationships and viewed factor analysis as a statistical multivariate analysis of those interrelationships. Urbina (2004) presented factor analysis development as a way to study test score relationships and the variety of available ability tests [not just spatial ability tests]. This discussion of factors supporting spatial ability concepts is based on factor analysis methodology used in graphics education research (Carroll, 1993; Cornoldi & Vecchi, 2003; Eliot, 2000; McGee, 1978). Bodner and Guay (1997) discussed two factors that emerged from spatial ability research: spatial orientation, which involves not being puzzled by changes in visual inputs, and spatial visualization, which involves the ability to manage visual input components. Eliot and Smith (1983) presented factors, such as spatial relations, in the context of mental rotation of objects, spatial orientation as the understanding of how an object would appear from a different perspective, and visualization from a surface development context. In his research, Juhel (1991) focused on three factors: spatial orientation, which determines how an object will appear from a different position; spatial visualization, which involves the mental transformation of an object; and speeded rotation, which is the mental rotation of objects. In agreement with Bodner and Guay (1997), McGee (1979) studied two factors: spatial visualization as the mental manipulation of visual inputs; and spatial orientation, the comprehension of components in a visual arrangement. In Strong and Smith’s (2002) review of spatial research, the primary concept of “spatial visualization” was viewed as the ability to execute difficult mental rearrangements of objects, whereas Eliot and Smith (1983) used the term “spatial ability” for the same meaning. Strong and Smith believed that spatial orientation and spatial visualization, as defined by McGee, are the main spatial factors. Consequently, some of the factors that have been identified by various graphics education researchers are spatial visualization, spatial relations, spatial orientation, spatial cognition, spatial intelligence, spatial ability, and visualization (Hartman & Bertoline, 2005; Martin-Dorta, Saorin, & Contero, 2008; Miller & Bertoline, 1991; Sorby, 1999a). In Eliot's (2000) and Eliot and Smith's (1983) in-depth historical review of factor analysis research, the number, name, and meaning of the identified factors depend on which specific research is reviewed as discussed above.

Scoring level of the spatial ability test was based on the following table;

S.No.	Score (x)	Category
1	$20 \leq x \leq 25$	High
2	$15 \leq x \leq 19$	Intermediate
3	$0 \leq x \leq 14$	Low

Research Methodology Population of the study

All the 650,569 boys' secondary school students at secondary level were constituted the population of the study (Govt. of Khyber Pakhtunkhwa, 2017).

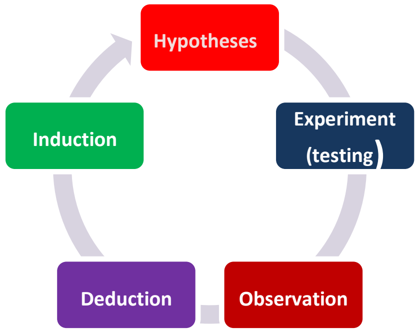
Sample of the study

Sixteen secondary/higher secondary schools were chosen (eight urban, and eight rural). From each sample school, twenty five students studying in 9th class were randomly (SRS) selected. In this way, 400 students as subjects were tested to investigate the spatial abilities (Farooq, R. A. & Tabassum, R., 2018; Crawford, 2008).

According to Gay, Mills, & Airasian (2012) if the population size is in thousands or in millions, then a sample size of 400 will be adequate.

Research Design

To conduct the study, An empirical research design was used.



Source: (Ahmad, 2017)

Research Instruments Material

Different cubes labeled geometric figures were used to observe spatial ability (five items of Mental Rotation), two glasses and a toy/body were used to observe spatial ability (five items of Perception of Spatial Position). Apart from the aforesaid ten items, fifteen items test (five items

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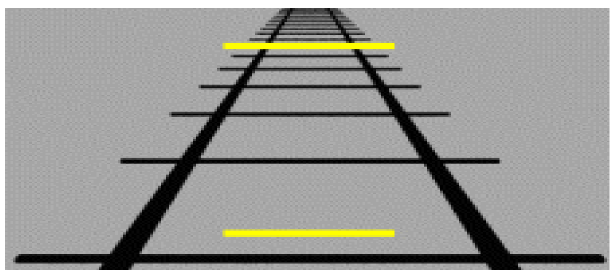
for each remaining three aspects of spatial ability i.e. perceptual constancy, perception of spatial relationships, and visual discrimination) were administered to collect data.

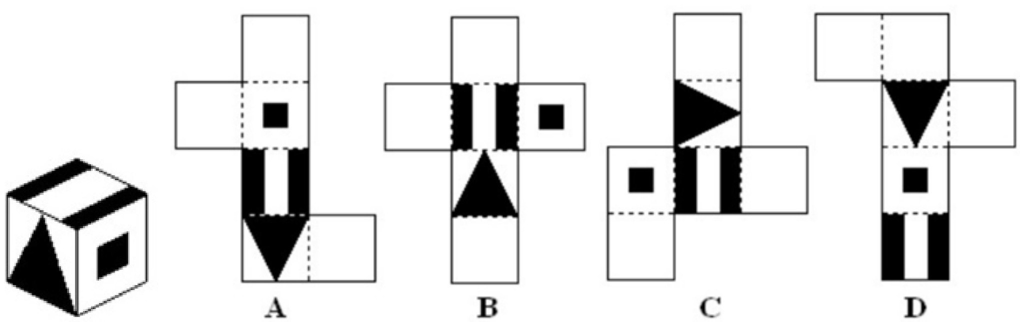
Procedure of the Study

The experiment was based on exploring spatial ability in geometry. The researcher conducted the experiment in the following way; all sampled schools were visited personally by the researcher. Twenty-five items test was administered to 25 randomly selected students in each visited schools through test/ observation. (Hubert Maier and other mathematicians have suggested five aspects of spatial ability). Following five aspects of spatial ability were tested;

(i). Perceptual constancy,

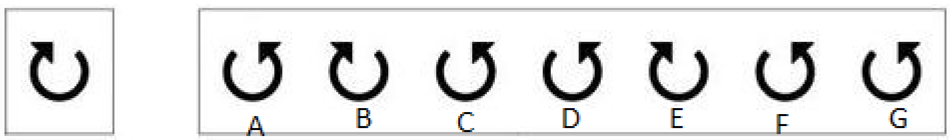


(ii). Mental rotation,

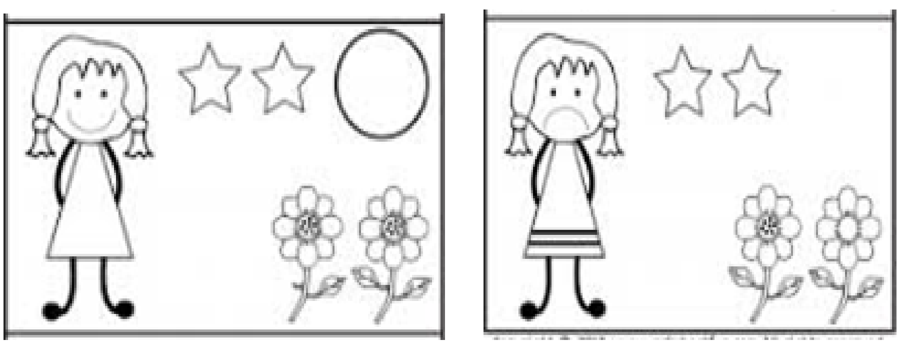


(iii). Perception of spatial position,

(iv). Perception of spatial relationship,



(v). Visual discrimination.



Data Collection

Primary data were collected through test/ observation in a way that fifteen items related to (a) Perceptual constancy, (b) perception of spatial relation, and (c) visual discrimination were collected through a test made of five items each and ten items related to (d) mental rotation, and (e) perception of spatial position were collected through observation from the sample. Curriculum document was consisted of secondary data.

Analyses of the Data

Collected data were analyzed in the following way;

i) Mean and standard Deviation of the marks/ observation sheet were calculated.

ii) In order to find out the significant difference among the rural and urban secondary schools' students, t-test was applied.

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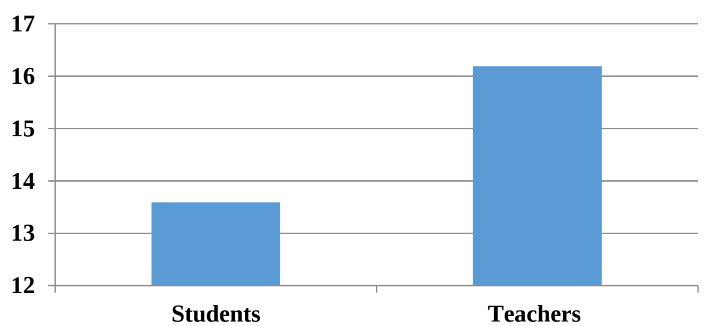
Table. Comparison of Spatial ability of teachers and students

	N	No. of schools	Total scores	Mean
Students	400	16	217.44	13.590
Teachers	16	16	259.00	16.187

Table shows that the means value (16.187) of teacher scores was greater than the means value (13.590) students' scores. It means that spatial ability of teachers was much better than the secondary school students.

Fig. Mean scores of spatial ability of school students and teachers



H₀: There is no significant difference between urban and rural secondary school students on spatial abilities.

Table. Comparison of spatial ability of urban and rural secondary school students

School Location	N	Mean	SD	t-value	p-value
Urban	200	13.8750	3.71462	1.516	0.130
Rural	200	13.3350	3.40304		

df= 398 Table value on 0.05 level = 2.011

Table shows that there was a non-significant difference between the urban and rural schools students regarding the spatial ability. The t calculated value 1.516 was less than table value of (2.011) at 0.05 level. There was no significant difference in the mean values of urban and rural schools students on spatial ability.

H_0: There is no significant difference between public and private secondary school students on spatial abilities.

Table . Comparison of spatial ability of public and private secondary school students (sum of 5 aspects)

School sector	N	Mean	SD	t-value	p-value
Public	200	12.4500	3.42288	6.835	.000
Private	200	14.7600	3.33587		

df= 398 Table value on 0.05 level = 2.011

Table shows that there was a significant difference between the public and private schools student regarding the spatial ability. The t calculated value 6.835 was greater than table value of (2.011) at 0.05 level of significance. The mean value regarding the spatial ability of the private (14.76) school students was greater that mean value (12.45) public school students.

Discussion

The study was carried out at observing the spatial ability of students and Mathematics teachers. The study was conducted in urban and rural areas of Khyber Pakhtunkhwa. The result of the study reveals that the performance of students and their Mathematics teachers regarding five aspects of perceptual constancy, mental rotation, perception of spatial position, perception of spatial relationship and visual discrimination was determined. Overall discussion of the study is as under:

The results of the study concluded that the spatial ability of mathematics teachers was greater (16.187) than mean value (13.590) students.

Recommendation

On the basis of conclusions, following recommendations were made; the study explored that there was a significance difference among all schools students' scores regarding spatial ability of the students. Comparably the spatial ability of private schools students with reference to the different aspects of spatial ability like perceptual consistency, mental rotation, perception of spatial position, perception of spatial relationship and visual discrimination was much better than the public schools students. National Council of Teachers in Mathematics (NCTM) standards can be helpful to revise the curriculum. After the revision of the curriculum the students will be able to use spatial ability tools while learning mathematics to increase their academic performance. It is recommended that in preservice and in-service training of the teachers the teachers may be trained to use spatial ability tools in their teaching. Due to this, the teachers will be able to use these spatial ability tools while teaching mathematics in real classes of their schools. In this way, the students' academic performance in the subject of mathematics will increase.

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