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Multi-objective optimization design of NAD-EWMA control chart based on vector autoregressive model

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Abstract

In this paper, we propose a multi-objective optimal design method based on non-parametric, adaptive and dynamic EWMA control charts for the problem of a small number of samples and uncertain distribution faced by designing quality control charts for multi-variety and small batch manufacturing processes. Based on nonparametric statistical theory and adaptive control, a control chart statistic independent of sample data distribution is constructed, and a dynamic sampling method based on clustering distance is designed to realize sample sampling. On this basis, a control chart multi-objective optimal design model is established considering statistical and economic aspects, and the model is solved based on vector autoregressive algorithm, and then a non-parametric adaptive dynamic EWMA control chart for the multi-variety small batch system is constructed. The experimental results show that the root mean square error of processing quality prediction of the proposed vector autoregressive model is reduced by 4.67% and 4.14%, respectively, compared with the effects of SVM and PSO-SVM quality prediction models. The control chart based on the vector autoregressive model proposed in this paper can quickly monitor the quality abnormalities with high monitoring performance, which provides an effective way to monitor the quality of the multi-species and small-lot manufacturing process.

Keywords: *quality control, nonparametric statistics, EWMA control chart, vector autoregression, PSO-SVM quality prediction model*

Introduction

Due to the increasingly diverse individual customer needs, the multi-variety, small-batch production method has become the dominant production mode for global companies. However, its many batches, few process quality data, and variable product parameters lead to quality control problems such as insufficient sample data, uncertain distribution, and repeated and variable process drift (Tang, Sun, Hu, & Castagliola, 2019) (Li, Xie, & Zhou, 2018) (Mukherjee & Marozzi, 2017). The traditional SPC control chart is a statistical control method based on sufficient sample data and assuming a normal distribution, which is difficult to solve the dynamic and variable quality control problems such as little sample data and uncertain distribution in the multi-variety small batch production mode (Hu, Castagliola, Zhou, & Tang, 2019) (Ping An 2021). Although there are

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improved methods based on grouping techniques and Bayesian, the quality control effects are far from the expected results. Therefore, constructing non-normal and small-sample-oriented quality control graphs is the key to improving the product quality of multi-batch and small-batch manufacturing processes and is also a current research difficulty in quality engineering.

The core problems of quality control in multi-variety small batch manufacturing processes are uncertain distribution, variable process drift, and small sample data. In addressing the problem of uncertain sample distribution, nonparametric statistical control charts have attracted extensive attention from scholars at home and abroad, and methods such as rank statistics and nonparametric tests have been used to realize the construction of nonparametric control charts in the case of unknown sample distribution (Chi, 2013) (Yang Ming 2020). In the literature (Wang Haiyu, 2021), a multivariate nonparametric control chart based on a high-dimensional rank test is proposed for the problem of unknown data distribution, high dimensionality and small samples, which is very robust in the case of non-normality and large drift. In the literature (Chen Keqiang, 2022), a nonparametric bi-uniform weighted moving average control chart based on the sign test statistic is proposed for the case of unknown distribution to achieve effective monitoring of smaller drifts. In the literature (Quinino, Ho, Cruz, & Bessegato, 2020) an online sequential detection method based on nonparametric tests and EWMA control charts is proposed for the two-sample problem with better robustness in the case of non-normal data. This shows that the nonparametric control chart can achieve drift monitoring of parameters such as position and scale under the condition of unknown sample distribution (Wiederhold, Greipel, Ottone, & Schmitt, 2016). To improve the detection efficiency of control charts for different drifts while taking into account the monitoring of large and small drifts, adaptive improvement of control charts is often performed. In the literature (Steiner, Lee Geyer, & Wesolowsky, 1996), a Markov chain model of MEWMA was developed using ARL as an indicator to analyze the main parameters affecting the statistical performance and to derive a method for parameter selection in different environments. In the literature (Nawaz, Raza, & Han, 2020), a Markov chain approach was used to achieve the economic design of the VSSIT2 control chart under an unknown covariance matrix using a genetic algorithm with the objective of minimum profit loss. The literature (Zhao & Driscoll, 2016) uses the average running chain length and the average additional secondary loss as effect indicators to determine the mapping strategy's complete dominance to monitor the manufacturing process's quality anomalies.

This paper proposes a nonparametric adaptive dynamic EWMA control chart for the non-normal, small-sample case. On the basis of cluster analysis of key processes to expand the sample data, a dynamic sampling method based on cluster distance is designed to realize the sampling of the data required for the control chart to achieve the parameter estimation of the sample. Next, a vector autoregressive model of the NAD-EWMA control chart is constructed, and then the calculation method of the statistical and economic evaluation indexes of the NAD-EWMA control chart is determined, and the method is simplified for the multi-variety small-lot manufacturing model. A

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multi-objective optimal design model of the NAD-EWMA control chart is established to support the subsequent multi-objective parameter optimization design. Then a cloud clear comprehensive evaluation method is proposed to realize the model objective function's linear weighting and the objective function's multi-objective processing. On this basis, for the problem that the step length in the autoregressive vector algorithm affects the algorithm's performance, the adaptive field of view and step length are adopted to improve the algorithm's performance, and the multi-objective solution of the model is realized. Finally, the actual production situation of a precision complex component manufacturing enterprise is used as an example, and the simulation results verify the effectiveness and feasibility of the method involved in this paper using Python language.

Non-parametric adaptive dynamic EWMA control chart

For the uncertainty of product quality in the multi-variety and small batch flexible manufacturing processes, based on nonparametric statistics, without making assumptions on the overall sample distribution. The parameter estimation of the actual sample data is realized to eliminate the errors in the assumptions of the model conditions by the traditional parametric methods to establish the quality control chart and realize the quality control of the manufacturing process (You, Shahrin, & Mustafa, 2021). Based on this, nonparametric statistical methods can adapt to complex build manufacturing processes with uncertain process data distribution, and the construction of nonparametric statistical control charts and parameter optimization are also key aspects of quality control in other multi-species and small-lot manufacturing fields. The non-parametric adaptive dynamic EWMA control chart based on the non-parametric adaptive dynamic EWMA control chart for the quality control of critical processes in multi-variety small batch manufacturing processes is shown in Figure 1 (Wang, 2016).

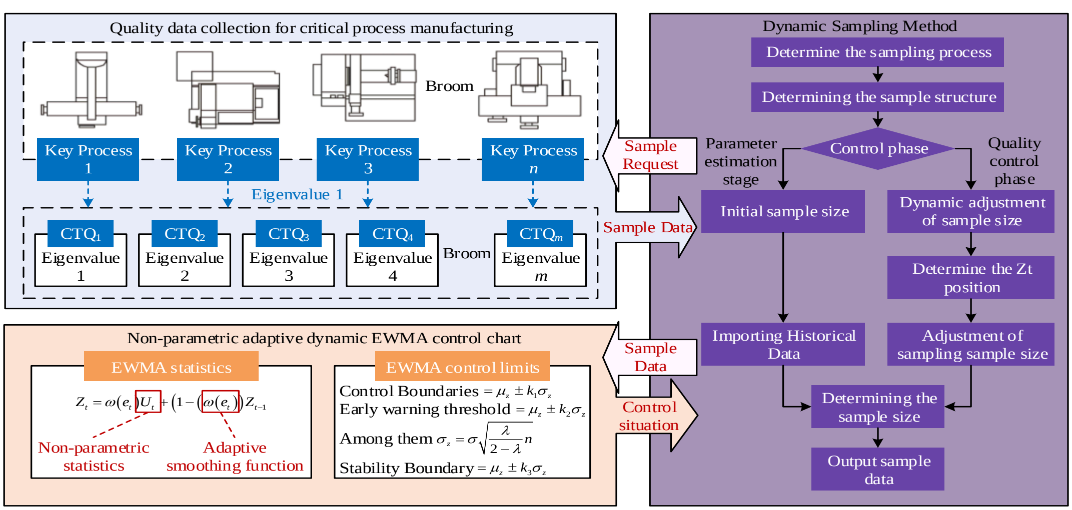

Figure 1: Quality control method for multi-variety small batch manufacturing process

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Non-parametric Adaptive Statistics

U statistic

Let the sample be a sample taken from the overall population X. Then the random variable U can be constructed as:

$$U = \frac{1}{n}\sum x_i \quad (1)$$

Where U is the random variable. x_i is the sample X amount at i.

Compared with the original sample, the expectation of this U statistic is still the mean of the original sample, but its variance will be reduced to $\frac{1}{n}$ of the original sample method. therefore, the probability density function of the sample will be changed, and the probability distribution of its statistic will be more concentrated in the sample mean, so the intersection area of the probability distribution density function will be reduced, which can reduce the probability of sample discrimination error.

Let the sample be taken from the overall X, h as an m-element function $(m < n)$, if it is an unbiased estimate of the overall distribution parameters:

$$U_n^{def} = \frac{1}{n(n-1)\cdots(n-m+1)} \sum_{1,,\ i_1 \neq i_2 \neq \cdots \neq i_m,,\ n} h\left(X_{i_1}, X_{i_2}, \cdots, X_{i_m}\right) \quad (2)$$

Where U_n^{def} is called the U statistic, or the sample-based U statistic with h as the kernel.

If the kernel function $h(X_1, X_2, \cdots, X_m) = \sum_{i=1}^{m} p_i X_i$ of $X_1, X_2, \cdots, X_m$ is given as an unbiased estimate of the overall mean μ. In particular when $m = 2$ and $p_1 = p_1 = \frac{1}{2}$, then:

$$U_n = \frac{2}{n(n-1)} \sum_{1,,\ i<j,,\ n} \frac{X_i + X_j}{2} = \bar{X} \quad (3)$$

Where U_n is an unbiased estimate of the overall mean and $\bar{X}$ is the sample mean.

Similarly, if the kernel function $h(X_i, X_j) = X_i^2 - X_i X_j$ for $X_1, X_2, \cdots, X_m$ is given, then $U_n = \frac{1}{n(n-1)} \sum_{i=1} \left(X_i^2 - X_i X_j\right)$ can be taken as an unbiased estimate of the overall variance σ^2, i.e:

$$U_n = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 = S^2 \qquad (4)$$

Where S^2 is the amount of unbiased estimation variance.

NAD-EWMA control chart statistics

In the manufacturing process of complex components, multi-variety, low-volume flexible mixed-flow production is mostly used, so the following assumptions are made:

Let the random variable X does not follow a normal distribution, and its sample distribution is unknown. Let its mean be denoted as μ and its variance as σ^2. When the process is controlled, the target value of the process mean is noted as $\mu = \mu_0$ and the standard deviation is noted as $\sigma = \sigma_0$; when the process is out of control, σ remains constant but $\mu = \mu_0 + \delta\sigma$, where δ is the fluctuation of the process mean, i.e., the process occurs only with a shift of the mean. Based on this hypothesis, the statistics of the EWMA control chart are constructed as follows.

Using the statistical measure method, the sample mean is estimated by substituting equation (2) into equation (4) to obtain equation (5), and the statistic for constructing the EWMA control chart with nonparametric statistics is:

$$Z_t = \lambda \bar{X} + (1-\lambda) Z_{t-1} \qquad (5)$$

$$Z_t = \lambda U_t + (1-\lambda) Z_{t-1} \qquad (6)$$

Where λ is the smoothing factor, and $\lambda \in (0,1)$, $Z_0 = \mu_0$.

In the multi-variety small batch manufacturing process, the sample drift size is difficult to determine due to its production characteristics, such as multi-variety cross-parallel, single-variety small batch, and production and trial production coexistence. If the statistical smoothing parameters are set artificially by the quality control practitioners directly, it will lead to a large error in the control process, significantly reducing the accuracy of monitoring the manufacturing process. Therefore, an equivalent smoothing function is introduced instead of smooth parameters to ensure effective manufacturing process control under different drifts in the control chart (Tavana, Li, Mobin, Komaki, & Teymourian, 2016). The nonparametric adaptive dynamic EWMA control chart (NAD-EWMA) with the statistics of:

$$Z_t = \omega(e_t) U_t + \left(1 - \omega(e_t)\right) Z_{t-1} \qquad (7)$$

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Where $\omega(e_t)$ is the equivalent smoothing function and $\omega(e_t)=\frac{\phi(e_t)}{e_t}$. Since the estimation term $e_t = U_t - Z_{t-1}$ of the process error, then we have:

$$\omega(e_t)=\begin{cases} \frac{e_t+(1-\lambda)\gamma}{e_t}=1+\frac{(1-\lambda)\gamma}{e_t}, e_t<-\gamma \\ \frac{\lambda e_t}{e_t}=\lambda, |e_t|,, \gamma \\ \frac{e_t-(1-\lambda)\gamma}{e_t}=1-\frac{(1-\lambda)\gamma}{e_t}, e_t>\gamma \end{cases} \quad (8)$$

Where e_t is the smoothing function and γ is the random error.

Obviously, the function value of the equivalent smoothing function $\omega(e_t)$ is directly related to the absolute value of the error term, while the threshold γ is used as the main parameter in the control chart design stage. It will directly affect the value of the equivalent smoothing function, which in turn affects the control chart performance. After the control chart design is completed, γ can be substituted as a constant into the equivalent smoothing function of the control chart.

EWMA control chart

Control Boundaries

To ensure that the control chart can better utilize the sample information and improve the control efficiency, the region within the control boundary of the control chart is divided into an early warning region and a stable region as shown in Figure 2. Therefore, the management bounds of the non-parametric adaptive dynamic EWMA control chart are as follows:

$$\begin{aligned} UCL &= \mu_0 + k_1\sigma_Z \quad & LCL &= \mu_0 - k_1\sigma_Z \\ UWL &= \mu_0 + k_2\sigma_Z \quad & LWL &= \mu_0 - k_2\sigma_Z \\ USL &= \mu_0 + k_3\sigma_Z \quad & LSL &= \mu_0 - k_3\sigma_Z \end{aligned} \quad (9)$$

Where, k_1, k_2, k_3 are the control limit coefficient, warning limit coefficient, stability limit coefficient, and $0<|k_3|<|k_2|<|k_1|$, take $k_2 = 2/3k_1, k_3 = 1/3k_1, \sigma_Z = \sigma\sqrt{\frac{\lambda}{2-\lambda}}$.

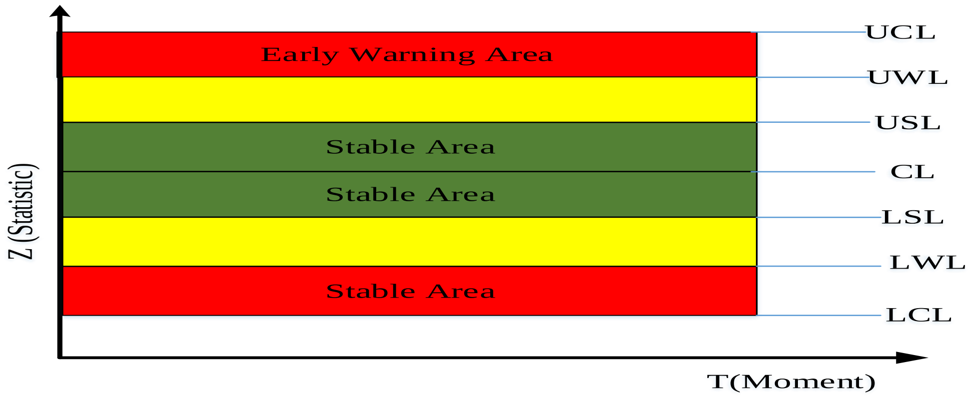

Figure 2: Control map control area division

Therefore, the NAD-EWMA control chart statistic Z_t will have three sampling scenarios, and if the statistic Z_t falls in the warning region, the process is considered to be about to get out of control, thus expanding the sample size to ensure that the statistic can obtain more completed estimates of the overall sample parameters. If the statistic Z_t falls between the warning region and the stable region, the sample capacity is not changed. If the statistic Z_t falls in the stable region, the process is considered stable, and the sample capacity can be reduced appropriately to ensure that the sample data of the reference process occupies a larger sample proportion, thus improving the control accuracy of the control chart.

Dynamic sampling method design

Control charts will generally be divided into dynamic and static control charts according to their sampling methods. Existing studies on dynamic sampling methods for control processes mainly focus on changing the sample capacity, sampling interval, and sampling time point (Cui, Geng, Zhu, & Han, 2017). However, the traditional sampling method is no longer applicable in the manufacturing process of multiple varieties and small batches, and a new data sampling method needs to be constructed to improve the monitoring efficiency of control charts.

In response to the characteristics of multi-variety, low-volume manufacturing processes, the production method of multi-variety mixed-flow production is mostly used, leading to the problem of small sample data size for a single variety and a large number of overall processes. Therefore, to expand the sample size, a sampling method with variable sample proportions was designed based on the use of key process identification and cluster analysis methods, using the clustering distance between the reference key process (the key process where quality control is implemented) and the clustered key process as an indicator (Yang, Guo, & Liao, 2012), with the following steps:

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Step 1: Select the reference key process and determine the required sample size N .

Step 2: Obtain the historical data sample size N_0 of the reference key process, and if $N_0 > N$, do not extract the sample data of the clustered key process. Otherwise, obtain the clustering distance of each clustering key process and determine the sample proportion of clustering processes.

Let $N_1, N_2, \cdots, N_i, \cdots, N_n$ be the theoretical sampling number of each clustering key process respectively, then the theoretical sampling ratio of clustering process is:

$$N_1 : \cdots : N_i : \cdots : N_n = d_1 : \cdots : d_i : \cdots : d_n \tag{10}$$

Where $d_1, d_2, \cdots, d_i \cdots, d_n$ is the clustering distance between the reference key process and each clustered key process.

Step 3: Calculate the theoretical sample size N_i for each clustering key process according to equation (10) as:

$$N_i = \left(N - N_0\right) \frac{d_i}{\sum_{i=1}^{n} d_i} \tag{11}$$

Where d_i is the clustering distance between the reference key process and the clustering key process at i .

Step 4: The sample size for each clustering key process is adjusted according to the actual historical data volume.

Multi-objective optimization model construction for control charts based on vector autoregression

The vector autoregressive model can analyze the relationship between product design parameters and target characteristics. Blocking can solve the problem when the attributes of the independent variables are inconsistent. Vector autoregression describes the relationship between product processing parameters of different magnitudes and product processing quality output in a quantitative manner. Using vector autoregression can handle incomplete data sets and retain all valid information about the product, and the steps of vector autoregression model construction are as follows (Amiri, Bashiri, Maleki, & Moghaddam, 2014).

Vector autoregression

Suppose the data set X, Y has m samples and n features, $X = \{x_1, x_2, ..., x_n\}$ is the independent variable, and $Y = \{y_1, y_2, ..., y_n\}$ is the dependent variable.

The n independent variables are divided into n blocks, and the missing independent variables are excluded before the regression, and the remaining data in the block are used as the regression of the independent variables with the dependent variable, and each block of independent variables is linearly regressed with the dependent variable in turn. Then the vector autoregressive model is:

$$Y^{(i)} = \hat{k}_i x_i + b_i + e_i \tag{12}$$

Where e_i is a random term.

Figure 3 shows the vector autoregressive process, where $Y^{(i)}$ denotes the linear regression model after fitting the i th feature $[x_i]$ to Y, $\hat{k}_i$ denotes the fitted regression coefficient as a constant term, and e_i denotes the random error. k_i measures the existence of a sample relationship between Y and k_i after excluding the effects of other factors. It is important for analyzing the influence of different processing parameters on the quality of the product output process. In addition, the idea of chunking can also solve, to a certain extent, the problem of complex data structure caused by many factors influencing product quality.

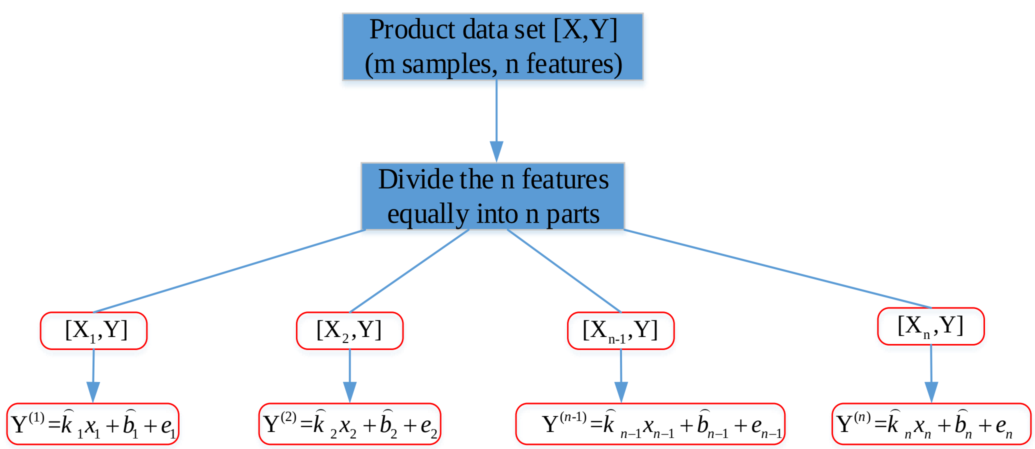

Figure 3: Vector autoregressive process

Kalman filtering data fusion

Data fusion is a technique that processes information from multiple sources to obtain the best

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estimate of the target object and its properties. The main data fusion methods are weighted average, Kalman filter, Bayesian estimation, D-S evidential inference, statistical decision theory, fuzzy logic, and neural network methods (Ganguly & Patel, 2019). Studies have shown that Kalman filtering can fuse information and extract accurate information amidst uncertainty. It also has superiority in dealing with linear relations. Therefore, this paper uses Kalman filtering for data fusion. Assume that Y is a random variable with a linear correlation with the variable X. Since Y is a random variable, for each determined value of X, Y has a distribution. Multiple distributions of Y are obtained by chunkwise regression, and the Kalman filter is used to fuse multiple distributions, and the distribution obtained after fusion is the optimal estimate of Y. Figure 4 shows $N_1(u_1, \sigma_1)$, $N_2(u_1, \sigma_1)$ representing two distributions of the same variable Y, respectively, using Kalman filter distribution fusion to obtain the distribution shown by the dashed line. The obtained distribution $N_i(u_i, \sigma_i)$ extracts the information of both distributions and is closer to the real situation of this variable.

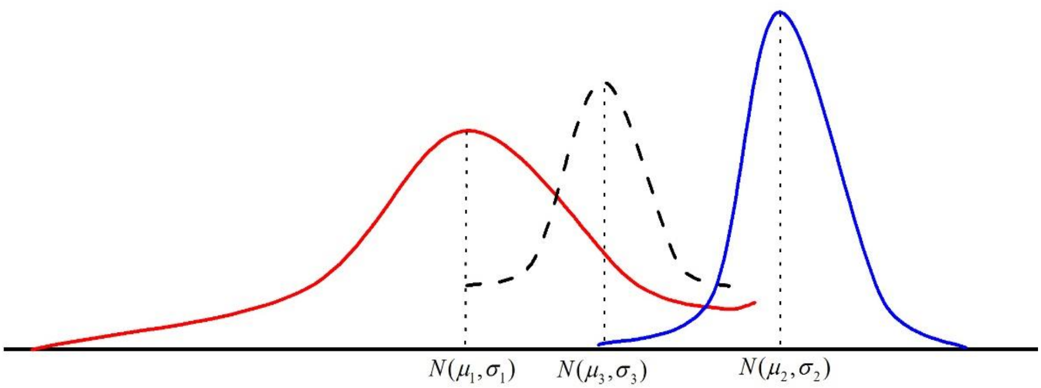

Figure 4: Kalman filtering data fusion process

Where u_3 and σ_3 are, respectively:

$$u_3 = \frac{u_1\sigma_2^2 + u_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2} = u_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}(u_2 - u_1) \qquad (13)$$

$$\sigma_3^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \sigma_1^2 - \frac{\sigma_1^4}{\sigma_1^2 + \sigma_2^2} \qquad (14)$$

As the distributions increase, the uncertainty of the information decreases. Equations (13) and (14) reveal that the mean of the fused distribution lies between the means of the two distributions before the merger, and the variance of the estimates of the fused variables becomes smaller, indicating that more accurate estimates can be achieved through data fusion.

Vector autoregressive model construction

In the multi-variety and small batch processing mode, one machine often needs to process many different specifications and products with different quality requirements simultaneously, and the historical processing data stored in each machine is extremely rich. Using these product history data, we can explore the common laws implied by the equipment and establish the quality prediction model of multi-variety and small batch processing for big industrial data when there is a new product processing demand, according to the processing quality prediction value of different equipment to select the optimal processing equipment to ensure the achievement of the product target processing quality.

In the multi-variety small batch product processing method, different product types and processing parameters make the data sets containing product quality information often have different degrees of deficiency. To solve the problem effectively, this paper constructs a processing quality prediction model based on chunking regression and Kalman filtering fusion for each intelligent processing machine. When the processing parameters are independent of each other, the chunking idea can effectively analyze the impact of each processing parameter on the quality of multi-variety small batch processing. The shortcomings in sample size expansion are compensated by discovering the potential patterns between product processing parameters and the output process quality.

First, the potential law between each processing parameter and the process quality output is analyzed by linear block regression and expressed as a distribution to solve the problem of missing information on the processing parameters. For any product to be processed, its processing quality is influenced by different processing parameters, and a distribution can describe each influence, and the Kalman filter is used to fuse the information of these distributions to form a new distribution, which is the distribution of product processing quality for a given combination of processing parameters. The new distribution is the distribution of the product processing quality for a given combination of processing parameters. The predicted value of the product processing quality and its confidence interval can be obtained from this distribution, and the appropriate processing route and suitable processing equipment can be selected according to the predicted result so that the product processing quality can be achieved.

The vector-based autoregressive processing equipment quality prediction model avoids the complex calculation of similarity compared to using similarity to expand the sample size. At the same time, the model analyzes how each processing parameter affects the product process output quality and solves the problem of insufficient interpretability of the processing quality prediction model based on machine learning algorithms. The specific steps of the model are as follows:

Step 1: Collect data. The quality information of the selected processing equipment's historical processed products, mainly includes the influencing factors of the product processing quality and

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the corresponding process quality.

Step 2: Data chunking. The set of influencing factors is chunked, and each influencing factor of processing quality is split into separate blocks, and each block contains only one influencing factor.

Step 3: Fitting linear regression. Linear regression is performed individually for each block of influencing factors and process quality to obtain multiple one-dimensional linear regression equations.

Step 4: Calculate the distribution parameters. Calculate the variance of the distribution of each one-dimensional linear regression equation.

Step 5: Prediction. Input the processing parameters of the product to be processed into the regression equation to obtain multiple process quality prediction results. The fourth step can obtain the distribution of multiple process quality prediction values, and the Kalman filter is used to fuse multiple distribution information.

Step 6: Output process quality point estimation and interval estimation. Calculate the maximum probability points in the fused process quality distribution and the 95% confidence interval.

Control chart multi-objective optimization validation results and analysis

A complex component manufacturing enterprise of an aerospace manufacturing company is a typical multi-variety and small-lot manufacturing mode enterprise whose product varieties and models are diverse, product lots are small, production processes are complex and unstable, and product quality cannot be guaranteed (Kavimani, Prakash, & Thankachan, 2019).

Through investigation and analysis, it is found that the main reason for the unstable product quality is that the quality management method currently adopted by the enterprise mainly relies on post-facto inspection, and the existing process control methods are not efficient enough and lack effective, stable and economical quality control methods. In this paper, based on the vector autoregressive control chart multi-objective optimization model construction method, four typical varieties of its complex frame class, crankshaft class, channel body class, and cylinder body class are used as examples to construct non-parametric adaptive dynamic EWMA control charts considering statistical and economic aspects to realize the quality control of its manufacturing process key processes, and then improve the product quality of this enterprise.

Initial parameter estimation

Taking the machining process of cylinder-like parts in complex components of aerospace manufacturing companies as an example, based on key process identification and cluster analysis methods, key processes and their clustering key processes are obtained and identified specifically, as shown in Table 1 and Table 2.

Table 1: Clustering distance and technical requirements for each key process

Key Processes	R1	C1	C2	C3	C4	C5
Clustering distance	-2.0425	2.1404	2.1028	1.7984	1.7125	1.4315
Technical requirements	$\phi 28.4_{0}^{+0.35}$	$\phi 45.8_{0}^{+0.183}$	$\phi 24_{0}^{+0.104}$	$\phi 18_{-0.15}^{+0.87}$	$\phi 18_{-0.127}^{+0.086}$	$\phi 8_{-0.024}^{+0.047}$
True Value	119.27	120.23	120.03	119.75	119.95	119.97
Predicted value	120.0012	120.1103	119.9987	120.0071	119.9928	119.9994
Relative error (%)	0.064	0.0311	0.0241	0.0161	0.0144	0.0406

Table 2: Reference key process and clustering key process production history data

Key Processes	**R1**	**C1**	**C2**	**C3**	**C4**	**C5**
1	27.664	45.14	31.087	27.994	8.06	17.929
2	27.679	45.102	31.641	27.903	8.022	17.919
3	27.801	45.155	31.392	27.96	7.977	17.908
4	27.72	45.176	31.807	28.078	7.912	17.991
5	27.771	45.166	31.516	27.957	7.914	17.846
6	27.781	45.035	31.353	28.098	8.087	17.951
7	27.823	45.078	31.494	27.906	7.964	17.944
8	27.799	45.117	31.799	27.935	——	17.842
9	27.839	——	31.504	28.021	——	17.977
10	27.631	——	31.768	——	——	17.924
11	——	——	31.232	——	——	17.921
12	——	——	31.739	——	——	17.853
13	——	——	31.861	——	——	——
14	——	——	31.203	——	——	——

Multi-objective optimal design analysis

Improved vector autoregressive optimization algorithm running on Windows 10 OS, Intel Xeon E3-1230 V3 CPU, 16G RAM desktop PC, implemented in Python. The autoregressive vector algorithm is improved by setting the maximum number of iterations $N_{iter} = 800$, the number of attempts $Try\_Number = 100$, the number of vector autoregressions $N_{fish} = 30$, and the crowding degree $\delta = 0.05$.

Based on this, the initial solution set is randomly generated, the Kalman filter model of the control chart is constructed according to the hypothesis condition $m = 100$, and its average running chain length is calculated according to Eq. (12), and then its APL can be obtained according to Eq. (13). On this basis, take $\theta = 0.01$, $\rho = 1$, $W = 1000$, according to the formula (14) can be obtained when controlled quality loss coefficient $C_1 = 0.31474$, cost and quality cost coefficient $C_2 = 0.32014$ when out of control, and then calculate statistical indicators and economics, as follows:

$$APL_1 = (N+h) \cdot \frac{P_m \cdot (I-R)^{-1}U}{(P_m \cdot (I-R)^{-1}U) - 1} \cdot U + \frac{(N+h)}{2} = 46.3729 \quad (15)$$

$$APC = \frac{C_1 \cdot \frac{1}{\theta} + C_2 APL_1 + W}{\frac{\rho}{\theta} + APL_1} = 7.1029 \quad (16)$$

The data were extracted for each key process and normalized using the relative tolerance transformation method; the results are shown in Table 3.

Table 3: Sampling data set after relative tolerance conversion

Key Processes	R1	C1	C2	C3	C4	C5
1-6	0.4475	0.3817	-0.0818	-0.3327	-0.0848	-0.1751
7-12	-0.2186	0.0063	-0.5541	-0.7919	-0.5571	-0.8091
13-18	0.4461	0.1162	0.4123	-0.5515	0.4123	0.9176
19-24	0.6132	0.2351	0.4517	0.6814	0.4539	-0.1426
25-30	-0.2487	0.0913	0.1129	-0.9017	0.9014	1.0001

Based on the vector autoregressive evaluation method, five expert groups were selected to evaluate the statistical and economic indicators. The obtained clear evaluation vector $W = \{2.8011, 2.9509\}$. According to each agreeing expert to grade the index evaluation results, $E_{x\alpha}$, En_α and μ_{ij} of the forward cloud generator are calculated, and 100 cloud drops (rating values) are randomly generated by using the inverse cloud generator, and their mean values are taken as quantitative values, and then the cloud value vector $\mu = \{0.4159, 0.9987\}$ is obtained.

Based on this, the fitness (objective function value) of this initial solution can be calculated $Y = \alpha \cdot APL_1 + \beta \cdot APC$. Based on this, the fitness of the non-inferior solution after the iteration of all initial solutions can be obtained, as shown in Table 4. The non-inferior solution with the smallest adaptation degree is selected as the optimal solution according to the promising small characteristic of the optimization model.

Table 4: Optimal solutions and corresponding index values for the iterative update of the algorithm

Key Processes	**Symbols**	**R1**	**C1**	**C2**	**C3**	**C4**	**C5**
Selection of new generation	$iter$	1	21	35	61	75	135
Sample size	N	1	17	35	69	93	135
Adjustment margin	□N	37	42	43	33	25	27
Control Boundary Factor	k_1	2.8013	2.7986	2.1403	2.9425	3.2143	1.1423
Equivalent smoothing coefficient	λ	0.0725	0.0816	0.0717	0.0613	0.0809	0.0608
Valence	γ	0.9356	1.3092	1.0936	1.2513	1.2239	1.0604
Statistical indicators	APL_1	59.6351	58.6991	61.7405	39.4059	33.7113	29.6706
Econometric indicators	APC	6.6034	6.1047	6.8915	6.9417	6.3029	6.5015
Objective function	Y	29.4243	29.0195	28.9038	28.7361	28.3068	27.9801

From the iterative control chart in Fig. 5, it can be found that the AFSA algorithm converges to a minimum value of 18.0129 at the beginning of 106 iterations. In comparison, the autoregressive vector algorithm converges to a minimum value of 15.1739 at 121 iterations. by comparing before and after the improvement. It can be found that the improvement of adaptive field of view and step size has achieved a certain effect. The convergence speed and iteration accuracy have been significantly improved. At the beginning of the iteration of the autoregressive vector algorithm, the runaway average product length APL_1 of the control chart fluctuates widely. This is due to the large fluctuations of the sampling data N at the beginning of the algorithm, and the fluctuations of N will cause the APL_1 to show larger fluctuations when the APL_1 is stable. After the algorithm starts to converge, the statistical performance of the control chart tends to stabilize as the N fluctuates less and gradually tends to stabilize, so the APL_1 fluctuation starts to stabilize and begins to fluctuate around the minimum value.

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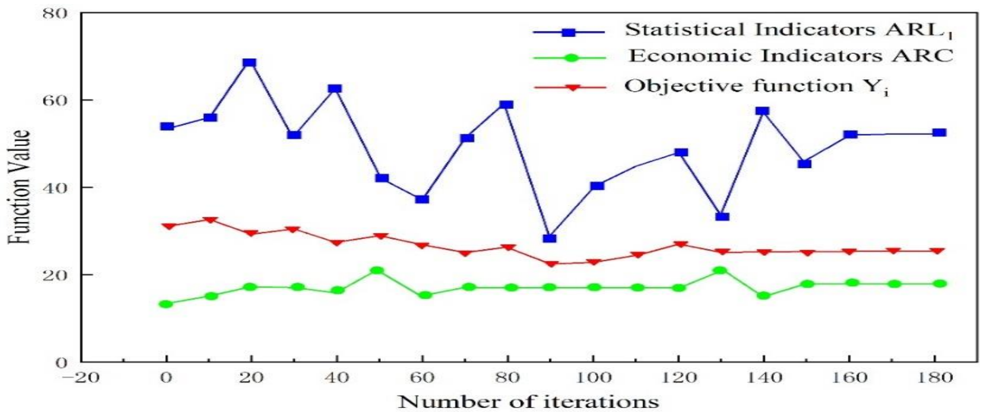

Figure 5: Iterative control chart of vector auto-regression algorithm

In addition, before and after optimization, although the APL_1 fluctuates significantly, while but the cost APC fluctuates less, which is mainly due to the performance of the control chart being more sensitive to its parameter changes. While in the multi-variety small batch manufacturing process, due to the small sampling interval, the cost of quality loss when its process is out of control is also relatively fixed, the economic cost of eliminating abnormalities and other aspects of the process is higher due to the higher cost, but the statistical and economic still have an optimization conflict. The optimized optimal solution ensures statistical performance improvement while also considering the control chart's implementation cost, which verifies the effectiveness of the optimal design method.

Quality control analysis of manufacturing process based on NAD-EWMA control chart

According to the control chart parameters in the obtained optimal solution X_{13} , the observed data of the manufacturing process of each key process are transformed and normalized. On this basis, sampling is performed according to the sampling method described in the text, and then the calculation results of the control chart statistics and control limits are completed, as shown in Table 5. and plot the corresponding NAD-EWMA control chart output results are shown in Fig. 6. The performance advantages and disadvantages of four control charts, NAD-EWMA control chart based on U statistic, ADF-EWMA control chart based on order, adaptive DF-EWMA control chart, and traditional EWMA control chart, are analyzed and compared by using the optimization design method proposed in this paper through Python language. In order not to lose generality, $APL_0 = 380$ is used as a criterion for discussion and analysis based on historical data information from the production process. The results show that the NAD-EWMA control chart based on U-statistic and the ADF-EWMA control chart based on order are robust in performance, and DF-EWMA is the next best. In contrast, EMWA is more economical due to its simple and convenient implementation but is less robust. Therefore, the optimal design of NAD-EWMA based on U-statistic has an improved effect. Therefore, the NAD-EWMA used in this paper has a better control effect in the production mode of multi-variety and small lot size, which is oriented to unknown sample distribution and variable process drift.

Table 5: Parameter values for each moment of the control chart control phase

Sampling moment	**Sample size** N	**Statistic** Z_t	**Equivalent smoothing function value** $\omega(e_t)$	**U statistic** U_t	**Variance** S_t^2	**Standard deviation** σ_z
1	60	0.1239	0.0596	0.1328	0.3109	0.0914
2	45	-0.0186	0.0842	-0.0382	0.3221	0.5051
3	15	-0.0181	0.0517	-0.0167	0.2618	0.0817
4	15	0.0126	0.6307	0.0361	0.2254	0.1532
5	15	0.0773	0.7718	0.0918	0.2965	0.3251
6	15	0.0871	0.3805	0.1023	0.2863	0.2751
7	15	0.0884	0.0517	0.0751	0.2755	0.0861
8	15	0.0712	0.0517	0.0523	0.2481	0.3053
9	15	0.0603	0.4923	0.0261	0.2021	0.3145
10	15	0.0453	0.5983	0.0198	0.1267	0.3546

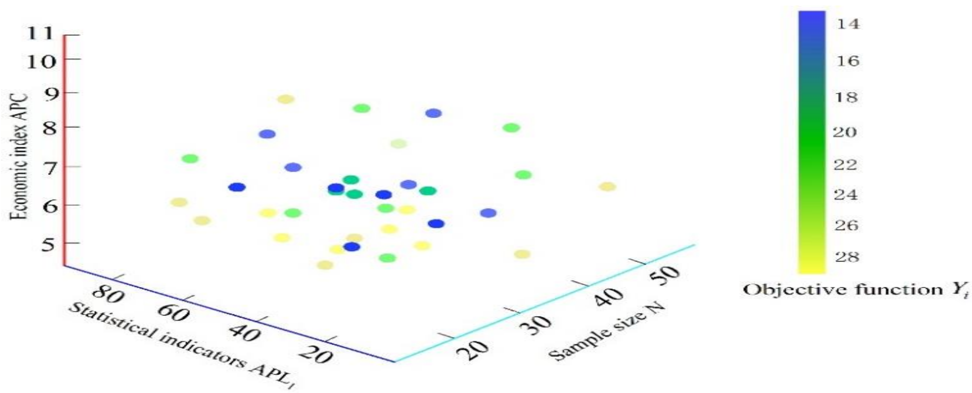

Figure 6: Comparison of the metrics of non-inferior solutions after algorithm iteration

Conclusion

In this paper, we analyze the product processing parameters and find that there are different degrees of correlations among the product processing parameters. This paper proposes a vector autoregression-based quality NAD-EWMA control chart prediction model for multi-species small batch product processing equipment based on this. Because of the natural advantages of SVM in small batch product processing quality prediction, the proposed vector autoregressive model is compared with the SVM-based processing quality prediction model and the PSO-SVM-based processing quality prediction model in terms of prediction effectiveness. Compared with the effects of SVM, and PSO-SVM quality prediction models, the root mean square error of processing quality prediction of the proposed vector autoregressive model is reduced by 4.67% and 4.14%, respectively. The proposed multi-species small batch processing quality prediction model has considerable advantages with high prediction accuracy and also gives confidence intervals for the production process quality prediction values based on the model to help companies make reasonable production decisions. The specific research findings are as follows:

(1) The U-statistic-based NAD-EWMA control chart and the order-based ADF-EWMA control chart perform robustly, with DF-EWMA being the next best. In contrast, EMWA is more economical due to its simple and convenient implementation but is less robust. Therefore, the optimal design of NAD-EWMA based on the U-statistic has an improved effect. Therefore, the NAD-EWMA used in this paper has a better control effect in the production mode of multi-variety and small lot size, which is oriented to unknown sample distribution and variable process drift.

(2) Interval estimation of process quality using the NAD-EWMA control chart prediction model proposed in this paper. The autoregressive vector algorithm-based multi-species small batch processing quality prediction model carries out the interval estimation of product processing quality. The simulation results show that the proposed model can give the upper and lower interval limits of product processing quality based on the error distribution more accurately, which can provide a decision basis for the scientific production planning of enterprises.

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